Approximation von Funktionen

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Satz von Taylor: Es sei I ein beliebiges Intervall und $x_0 \in I$ im Innern von I. f sei $(n+1)$-mal differenzierbar auf I. Dann gilt:

$$f(x) = \sum_{i=0}^{n} \frac{f^{(i)}(x_0)}{i!}(x-x_0)^i + R_n(x,x_0) \quad \text{mit} \quad R_n(x,x_0) = \frac{1}{n!}\int_{x_0}^{x} (x-t)^n f^{(n+1)}(t)\, dt$$

TAYLOR-Entwicklungen einiger wichtiger Funktionen:

$$e^x = 1 + \tfrac{1}{1!}x + \tfrac{1}{2!}x^2 + \tfrac{1}{3!}x^3 + \ldots = \sum_{n=0}^{\infty} \frac{x^n}{n!} \quad ; \quad x \in \mathbb{R}$$

$$\sin x = x - \tfrac{1}{3!}x^3 + \tfrac{1}{5!}x^5 - \tfrac{1}{7!}x^7 + \ldots = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} \quad ; \quad x \in \mathbb{R}$$

$$\cos x = 1 - \tfrac{1}{2!}x^2 + \tfrac{1}{4!}x^4 - \tfrac{1}{6!}x^6 + \ldots = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} \quad ; \quad x \in \mathbb{R}$$

$$\ln(1+x) = x - \tfrac{1}{2}x^2 + \tfrac{1}{3}x^3 - \tfrac{1}{4}x^4 + \ldots = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^n}{n} \quad ; \quad -1 < x \le 1$$

$$\ln(1-x) = -(x + \tfrac{1}{2}x^2 + \tfrac{1}{3}x^3 + \tfrac{1}{4}x^4 + \ldots) = -\sum_{n=1}^{\infty} \frac{x^n}{n} \quad ; \quad -1 \le x < 1$$

$$\ln\left(\tfrac{1+x}{1-x}\right) = 2(x + \tfrac{1}{3}x^3 + \tfrac{1}{5}x^5 + \tfrac{1}{7}x^7 + \ldots) = 2\sum_{n=0}^{\infty} \frac{x^{2n+1}}{2n+1} \quad ; \quad -1 < x < 1$$

$$\sinh x = x + \tfrac{1}{3!}x^3 + \tfrac{1}{5!}x^5 + \tfrac{1}{7!}x^7 + \ldots = \sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)!} \quad ; \quad x \in \mathbb{R}$$

$$\cosh x = 1 + \tfrac{1}{2!}x^2 + \tfrac{1}{4!}x^4 + \tfrac{1}{6!}x^6 + \ldots = \sum_{n=0}^{\infty} \frac{x^{2n}}{(2n)!} \quad ; \quad x \in \mathbb{R}$$

$$\arctan x = x - \tfrac{1}{3}x^3 + \tfrac{1}{5}x^5 - \tfrac{1}{7}x^7 + \ldots = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1} \quad ; \quad -1 \le x \le 1$$

$$(1+x)^\alpha = 1 + \alpha x + \tfrac{\alpha\cdot(\alpha-1)}{1\cdot 2}x^2$$

$$+\tfrac{\alpha\cdot(\alpha-1)\cdot(\alpha-2)}{1\cdot 2\cdot 3}x^3 + \ldots \quad ; \quad \alpha \ne 0$$

$$= \tbinom{\alpha}{0} + \tbinom{\alpha}{1}x + \tbinom{\alpha}{2}x^2 + \tbinom{\alpha}{3}x^3 + \ldots = \sum_{n=0}^{\infty} \binom{\alpha}{n} x^n \quad ; \quad -1 < x < 1$$